

SPRING LOADED SEE SAW

```
In[ ]:= Quit[];
```

```
(*****  
(*FUNCTIONS*)  
SkewSym[w_] :=  
  {{0, -w[[3, 1]], w[[2, 1]]}, {w[[3, 1]], 0, -w[[1, 1]]}, {-w[[2, 1]], w[[1, 1]], 0}};  
unSkewSym[V_] := {{V[[3, 2]]}, {V[[1, 3]]}, {V[[2, 1]]}}
```

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(*Functions from the MR Library *)
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```
VectToSE3[V_] :=  
  ArrayFlatten[{{VecToso3[{V[[1 ;; 3, 1]]}^T], {V[[4 ;; 6, 1]]}^T}, {0, 0}}]  
SE3ToVec[se3mat_] := {{se3mat[[3, 2]], se3mat[[1, 3]],  
  se3mat[[2, 1]], se3mat[[1, 4]], se3mat[[2, 4]], se3mat[[3, 4]]}^T  
TransInv[T_] := ArrayFlatten[{{T[[1 ;; 3, 1 ;; 3]]^T,  
  -T[[1 ;; 3, 1 ;; 3]]^T.T[[1 ;; 3, 4]]^T}, {0, 1}}]
```

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(*****
```

(*Transforms*)

```
(*SQUARE1 COM*)
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```
q = {{x1[t]}, {y1[t]}, {θ1[t]}, {x2[t]}, {y2[t]}, {θ2[t]}, {θ3[t]}};  
gWSq1 = {{Cos[θ1[t]], -Sin[θ1[t]], 0, x1[t]},  
  {Sin[θ1[t]], Cos[θ1[t]], 0, y1[t]}, {0, 0, 1, 0}, {0, 0, 0, 1}};
```

```
Print["gWSq1 = " MatrixForm[gWSq1]]
```

```
(*SQUARE2 COM*)
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```
gWSq2 = {{Cos[θ2[t]], -Sin[θ2[t]], 0, x2[t]},  
  {Sin[θ2[t]], Cos[θ2[t]], 0, y2[t]}, {0, 0, 1, 0}, {0, 0, 0, 1}};
```

```
Print["gWSq2 = " MatrixForm[gWSq2]]
```

```
TL = {-1.5, 1.5, 0, 1};
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```
BL = {-1.5, -1.5, 0, 1};
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```
BR = {1.5, -1.5, 0, 1};
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```
TR = {1.5, 1.5, 0, 1};
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(*SEE SAW*)
gWMid = {{Cos[θ3[t]], -Sin[θ3[t]], 0, 0},
        {Sin[θ3[t]], Cos[θ3[t]], 0, 6}, {0, 0, 1, 0}, {0, 0, 0, 1}};
Print["gWSSMid = " MatrixForm[gWMid]]
gMidSend1 = {{1, 0, 0, -15}, {0, 1, 0, -15 * Tan[θ3[t]]}, {0, 0, 1, 0}, {0, 0, 0, 1}};
Print["gMidWSSend1 = " MatrixForm[gMidSend1]]
gMidSend2 = {{1, 0, 0, 15}, {0, 1, 0, 15 * Tan[θ3[t]]}, {0, 0, 1, 0}, {0, 0, 0, 1}};
Print["gMidWSSend2 = " MatrixForm[gMidSend2]]
gWS1 = gWMid.gMidSend1;
gWS2 = gWMid.gMidSend2;
Print["gWSSend1 = gWSSMid.gMidWSSend1"]
Print["gWSSend2 = gWSSMid.gMidWSSend2"]

```

(*parameters*)

```

g = 9.8;
(*Square1*)
m1 = 5;
J1 = 1;
(*Square2*)
m2 = 2;
J2 = 1;
(*Seesaw*)
ms = 8;
Js = 1;
(*Springs*)
k1 = 6; (*MAX 12, MIN 3, RECOMMENDED 6*)
k2 = 6; (*MAX 12, MIN 3, RECOMMENDED 6*)

```

```

(*FIRST AND SECOND TIME DERIVATIVES*)
dq = D[q, t];
ddq = D[dq, t];

```

(*LAGRANGIAN*)

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Sq1MassInertiaMatrix = {{m1, 0, 0, 0, 0, 0}, {0, m1, 0, 0, 0, 0},
                        {0, 0, m1, 0, 0, 0}, {0, 0, 0, J1, 0, 0}, {0, 0, 0, 0, J1, 0}, {0, 0, 0, 0, 0, J1}};
Sq2MassInertiaMatrix = {{m2, 0, 0, 0, 0, 0}, {0, m2, 0, 0, 0, 0},
                        {0, 0, m2, 0, 0, 0}, {0, 0, 0, J2, 0, 0}, {0, 0, 0, 0, J2, 0}, {0, 0, 0, 0, 0, J2}};
SSMassInertiaMatrix = {{ms, 0, 0, 0, 0, 0}, {0, ms, 0, 0, 0, 0}, {0, 0, ms, 0, 0, 0},
                        {0, 0, 0, Js, 0, 0}, {0, 0, 0, 0, Js, 0}, {0, 0, 0, 0, 0, Js}};
(*SQUARE1*)
Sq1Twist = SE3ToVec[(TransInv[gWSq1].D[gWSq1, t])];

```

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KESq1 = (1/2) * (Sq1TwistT.Sq1MassInertiaMatrix.Sq1Twist);
VSq1 = m1 * g * gWSq1[[2, 4]];
LSq1 = KESq1 - VSq1;

(*SQUARE2*)
Sq2Twist = SE3ToVec[(TransInv[gWSq2].D[gWSq2, t])];
KESq2 = (1/2) * (Sq2TwistT.Sq2MassInertiaMatrix.Sq2Twist);
VSq2 = m2 * g * gWSq2[[2, 4]];
LSq2 = KESq2 - VSq2;

(*SEE SAW*)
SSTwist = SE3ToVec[(TransInv[gWMid].D[gWMid, t])];
KESS = (1/2) * (SSTwistT.SSMassInertiaMatrix.SSTwist);
VSS = ms * g * gWMid[[2, 4]];
ΔSpring1 = Sqrt[((gWS1[[1, 4]] - 15) ^ 2 + ((gWS1[[2, 4]] - 6) ^ 2));
ΔSpring2 = Sqrt[((gWS2[[1, 4]] - 15) ^ 2 + ((gWS2[[2, 4]] - 6) ^ 2));
VSpring = (1/2) * k1 * (ΔSpring1) ^ 2 + (1/2) * k2 * (ΔSpring2) ^ 2;
LSS = KESS - VSS - VSpring;

L = LSq1 + LSq2 + LSS;

```

(*CONSTRAINTS*)

```

φ1 = 15 * Cos[θ3[t]] - ((gWSq2.BL)[[1]]);
φ2 = 6 - 15 * Sin[θ3[t]] - ((gWSq2.BL)[[2]]);
(*((gWSq2.BL)[[2]] + (1.5 * Tan[θ3[t]])) - gWS1[[2,4]]);*)
φ3 = θ2[t] - θ3[t];

```

(*IMPACT*)

```

φ4 := (*Sqrt[((gWS2[[1,4]] - (x1[t] + 1.5)) ^ 2 + ((gWS2[[2,4]] - (y1[t] - 1.5)) ^ 2)*)
  ((y1[t] - 1.5) - (gWS2[[2, 4]]));
φ5 := (*Sqrt[((gWS1[[1,4]] - (x2[t] + 1.5)) ^ 2 + ((gWS1[[2,4]] - (y2[t] - 1.5)) ^ 2)*)
  ((y2[t] - 1.5) - (gWS1[[2, 4]]));

```

```

(*SQUARE1*)
Eq1 = D[D[L, x1'[t]], t] - D[L, x1[t]] ==
  λ1[t] * D[φ1, x1[t]] + λ2[t] * D[φ2, x1[t]] + λ3[t] * D[φ3, x1[t]];
Eq2 = D[D[L, y1'[t]], t] - D[L, y1[t]] ==
  λ1[t] * D[φ1, y1[t]] + λ2[t] * D[φ2, y1[t]] + λ3[t] * D[φ3, y1[t]];
Eq3 = D[D[L, θ1'[t]], t] - D[L, θ1[t]] ==
  λ1[t] * D[φ1, θ1[t]] + λ2[t] * D[φ2, θ1[t]] + λ3[t] * D[φ3, θ1[t]];

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(*SQUARE2*)
Eq4 = D[D[L, x2'[t]], t] - D[L, x2[t]] ==

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λ1[t] * D[φ1, x2[t]] + λ2[t] * D[φ2, x2[t]] + λ3[t] * D[φ3, x2[t]];
Eq5 = D[D[L, y2'[t]], t] - D[L, y2[t]] ==
λ1[t] * D[φ1, y2[t]] + λ2[t] * D[φ2, y2[t]] + λ3[t] * D[φ3, y2[t]];
Eq6 = D[D[L, θ2'[t]], t] - D[L, θ2[t]] ==
λ1[t] * D[φ1, θ2[t]] + λ2[t] * D[φ2, θ2[t]] + λ3[t] * D[φ3, θ2[t]];

(*SEE SAW*)
Eq7 = D[D[L, θ3'[t]], t] - D[L, θ3[t]] ==
λ1[t] * D[φ1, θ3[t]] + λ2[t] * D[φ2, θ3[t]] + λ3[t] * D[φ3, θ3[t]];
Eq8 = D[D[φ1, t], t] == 0;
Eq9 = D[D[φ2, t], t] == 0;
Eq10 = D[D[φ3, t], t] == 0;

(*Initialising config variable updates*)
x1n = Piecewise[{{0, t > 0 && t < 0}}];
y1n = Piecewise[{{0, t > 0 && t < 0}}];
θ1n = Piecewise[{{0, t > 0 && t < 0}}];
x2n = Piecewise[{{0, t > 0 && t < 0}}];
y2n = Piecewise[{{0, t > 0 && t < 0}}];
θ2n = Piecewise[{{0, t > 0 && t < 0}}];
θ3n = Piecewise[{{0, t > 0 && t < 0}}];

ELtemp = Solve[Eq1 && Eq2 && Eq3 && Eq4 && Eq5 && Eq6 && Eq7 && Eq8 && Eq9 && Eq10, {x1'[t],
y1'[t], θ1'[t], x2'[t], y2'[t], θ2'[t], θ3'[t], λ1[t], λ2[t], λ3[t]};
EL = {x1'[t] == ELtemp[[1, 1, 2]], y1'[t] == ELtemp[[1, 2, 2]],
θ1'[t] == ELtemp[[1, 3, 2]], x2'[t] == ELtemp[[1, 4, 2]], y2'[t] ==
ELtemp[[1, 5, 2]], θ2'[t] == ELtemp[[1, 6, 2]], θ3'[t] == ELtemp[[1, 7, 2]]};
InitCon = {x1[0] == 13.5, y1[0] == 35, x1'[0] == 0, y1'[0] == 0,
θ1'[0] == 0, θ1[0] == 0, x2[0] == -13.5, y2[0] == 7.7, x2'[0] == 0,
y2'[0] == 0, θ2'[0] == 0, θ2[0] == 0, θ3'[0] == 0, θ3[0] == -(0)};

(*HAMILTONAIN*)
p = D[L, dq];
H = {p}.dq - L;

(*Solve the equations of motion before impact*)
sol = NDSolve[Join[EL, InitCon], {x1[t], y1[t], θ1[t], x2[t], y2[t], θ2[t], θ3[t]},
{t, 0, 10}, Method -> {"EventLocator", "Event" -> ((y1[t] - 1.5) - (gWS1[[2, 4]])),
"EventAction" -> Throw[tmax = t, "StopIntegration"]}];
Print["The impact is at time ", tmax]

(*Updating variables leading upto impact*)
x1n = Piecewise[{{x1n, 0 ≤ t ≤ 0}, {sol[[1, 1, 2]], t > 0 && t < tmax}}];
y1n = Piecewise[{{y1n, 0 ≤ t ≤ 0}, {sol[[1, 2, 2]], t > 0 && t < tmax}}];
θ1n = Piecewise[{{θ1n, 0 ≤ t ≤ 0}, {sol[[1, 3, 2]], t > 0 && t < tmax}}];

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x2n = Piecewise[{{x2n, 0 ≤ t ≤ 0}, {sol[[1, 4, 2]], t > 0 && t < tmax}}];
y2n = Piecewise[{{y2n, 0 ≤ t ≤ 0}, {sol[[1, 5, 2]], t > 0 && t < tmax}}];
θ2n = Piecewise[{{θ2n, 0 ≤ t ≤ 0}, {sol[[1, 6, 2]], t > 0 && t < tmax}}];
θ3n = Piecewise[{{θ3n, 0 ≤ t ≤ 0}, {sol[[1, 7, 2]], t > 0 && t < tmax}}];
tmax1 = tmax;(*TIME OF IMPACT - Important*)

```

(*IMPACT LAWS*)

```
(*ELASTIC*)
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```
(*Hplus =
```

```

H/.sol/.{x1'[t] → x1plus,y1'[t] → y1plus,θ1'[t]→ θ1plus, x2'[t] → x2plus,
y2'[t] → y2plus, θ2'[t]→ θ2plus, θ3'[t]→ θ3plus,θ4'[t]→ θ4plus}/. t→ tmax;

```

```
Hminus = H/.sol/.{x1'[t] →D[x1[t]/.sol,t],y1'[t] →D[y1[t]/.sol,t],
```

```
θ1'[t]→ D[θ1[t]/.sol,t],x2'[t] →D[x2[t]/.sol,t],
```

```
y2'[t] →D[y2[t]/.sol,t],θ2'[t]→ D[θ2[t]/.sol,t],
```

```
θ3'[t]→ D[θ3[t]/.sol,t],θ4'[t]→ D[θ4[t]/.sol,t]}/. t→ tmax;
```

```
EQ1 = (Flatten[Hplus]-Flatten[Hminus])[[1]] == 0;*)
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(*****
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```
*****)(*PLASTIC*)
```

```
plastic = (D[φ4, q'] . dq) [[1]] /. sol /.
```

```
{x1'[t] → x1plus, y1'[t] → y1plus, θ1'[t] → θ1plus, x2'[t] → x2plus,
```

```
y2'[t] → y2plus, θ2'[t] → θ2plus, θ3'[t] → θ3plus} /. t → tmax;
```

```
EQ1 = Flatten[plastic] == 0;
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(*****
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*****)
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EQ2 =
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Flatten[p[[1, 1, 1]] /. sol /. {x1'[t] → x1plus, y1'[t] → y1plus, θ1'[t] → θ1plus,
x2'[t] → x2plus, y2'[t] → y2plus, θ2'[t] → θ2plus,
θ3'[t] → θ3plus} /. t → tmax] [[1]] -

```

```
Flatten[p[[1, 1, 1]] /. sol /. {x1'[t] → D[x1[t] /. sol, t],
```

```
y1'[t] → D[y1[t] /. sol, t], θ1'[t] → D[θ1[t] /. sol, t],
```

```
x2'[t] → D[x2[t] /. sol, t], y2'[t] → D[y2[t] /. sol, t],
```

```
θ2'[t] → D[θ2[t] /. sol, t], θ3'[t] → D[θ3[t] /. sol, t]} /. t → tmax] [[
```

```
1]] == λ * (D[φ4, x1[t]] + D[φ5, x1[t]]) /. sol /. t → tmax;
```

```
EQ3 = Flatten[p[[1, 1, 2]] /. sol /. {x1'[t] → x1plus, y1'[t] → y1plus,
```

```
θ1'[t] → θ1plus, x2'[t] → x2plus, y2'[t] → y2plus,
```

```
θ2'[t] → θ2plus, θ3'[t] → θ3plus} /. t → tmax] [[1]] -
```

```
Flatten[p[[1, 1, 2]] /. sol /. {x1'[t] → D[x1[t] /. sol, t],
```

```
y1'[t] → D[y1[t] /. sol, t], θ1'[t] → D[θ1[t] /. sol, t],
```

```
x2'[t] → D[x2[t] /. sol, t], y2'[t] → D[y2[t] /. sol, t],
```

```
θ2'[t] → D[θ2[t] /. sol, t], θ3'[t] → D[θ3[t] /. sol, t]} /. t → tmax] [[
```

```
1]] == λ * (D[φ4, y1[t]] + D[φ5, y1[t]]) /. sol /. t → tmax;
```

```
EQ4 = Flatten[p[[1, 1, 3]] /. sol /. {x1'[t] → x1plus, y1'[t] → y1plus,
```

```
θ1'[t] → θ1plus, x2'[t] → x2plus, y2'[t] → y2plus,
```

```

    \theta_2'[t] \to \theta_{2plus}, \theta_3'[t] \to \theta_{3plus} \} /. t \to tmax][[1]] -
  Flatten[p[[1, 1, 3]] /. sol /. {x1'[t] \to D[x1[t] /. sol, t],
    y1'[t] \to D[y1[t] /. sol, t], \theta_1'[t] \to D[\theta_1[t] /. sol, t],
    x2'[t] \to D[x2[t] /. sol, t], y2'[t] \to D[y2[t] /. sol, t], \theta_2'[t] \to
      D[\theta_2[t] /. sol, t], \theta_3'[t] \to D[\theta_3[t] /. sol, t]} /. t \to tmax][[1]] ==
  \lambda * (D[\phi_4, \theta_1[t]] + D[\phi_5, \theta_1[t]]) /. sol /. t \to tmax;
EQ5 = Flatten[p[[1, 1, 4]] /. sol /. {x1'[t] \to x1plus, y1'[t] \to y1plus,
  \theta_1'[t] \to \theta_{1plus}, x2'[t] \to x2plus, y2'[t] \to y2plus,
  \theta_2'[t] \to \theta_{2plus}, \theta_3'[t] \to \theta_{3plus} \} /. t \to tmax][[1]] -
  Flatten[p[[1, 1, 4]] /. sol /. {x1'[t] \to D[x1[t] /. sol, t],
    y1'[t] \to D[y1[t] /. sol, t], \theta_1'[t] \to D[\theta_1[t] /. sol, t],
    x2'[t] \to D[x2[t] /. sol, t], y2'[t] \to D[y2[t] /. sol, t],
    \theta_2'[t] \to D[\theta_2[t] /. sol, t], \theta_3'[t] \to D[\theta_3[t] /. sol, t]} /. t \to tmax][[
    1]] == \lambda * (D[\phi_4, x2[t]] + D[\phi_5, x2[t]]) /. sol /. t \to tmax;
EQ6 = Flatten[p[[1, 1, 5]] /. sol /. {x1'[t] \to x1plus, y1'[t] \to y1plus,
  \theta_1'[t] \to \theta_{1plus}, x2'[t] \to x2plus, y2'[t] \to y2plus,
  \theta_2'[t] \to \theta_{2plus}, \theta_3'[t] \to \theta_{3plus} \} /. t \to tmax][[1]] -
  Flatten[p[[1, 1, 5]] /. sol /. {x1'[t] \to D[x1[t] /. sol, t],
    y1'[t] \to D[y1[t] /. sol, t], \theta_1'[t] \to D[\theta_1[t] /. sol, t],
    x2'[t] \to D[x2[t] /. sol, t], y2'[t] \to D[y2[t] /. sol, t],
    \theta_2'[t] \to D[\theta_2[t] /. sol, t], \theta_3'[t] \to D[\theta_3[t] /. sol, t]} /. t \to tmax][[
    1]] == \lambda * (D[\phi_4, y2[t]] + D[\phi_5, y2[t]]) /. sol /. t \to tmax;
EQ7 = Flatten[p[[1, 1, 6]] /. sol /. {x1'[t] \to x1plus, y1'[t] \to y1plus,
  \theta_1'[t] \to \theta_{1plus}, x2'[t] \to x2plus, y2'[t] \to y2plus,
  \theta_2'[t] \to \theta_{2plus}, \theta_3'[t] \to \theta_{3plus} \} /. t \to tmax][[1]] -
  Flatten[p[[1, 1, 6]] /. sol /. {x1'[t] \to D[x1[t] /. sol, t],
    y1'[t] \to D[y1[t] /. sol, t], \theta_1'[t] \to D[\theta_1[t] /. sol, t],
    x2'[t] \to D[x2[t] /. sol, t], y2'[t] \to D[y2[t] /. sol, t],
    \theta_2'[t] \to D[\theta_2[t] /. sol, t], \theta_3'[t] \to D[\theta_3[t] /. sol, t]} /. t \to tmax][[
    1]] == \lambda * (D[\phi_4, \theta_2[t]] + D[\phi_5, \theta_2[t]]) /. sol /. t \to tmax;
EQ8 = Flatten[p[[1, 1, 7]] /. sol /. {x1'[t] \to x1plus, y1'[t] \to y1plus,
  \theta_1'[t] \to \theta_{1plus}, x2'[t] \to x2plus, y2'[t] \to y2plus,
  \theta_2'[t] \to \theta_{2plus}, \theta_3'[t] \to \theta_{3plus} \} /. t \to tmax][[1]] -
  Flatten[p[[1, 1, 7]] /. sol /. {x1'[t] \to D[x1[t] /. sol, t],
    y1'[t] \to D[y1[t] /. sol, t], \theta_1'[t] \to D[\theta_1[t] /. sol, t],
    x2'[t] \to D[x2[t] /. sol, t], y2'[t] \to D[y2[t] /. sol, t],
    \theta_2'[t] \to D[\theta_2[t] /. sol, t], \theta_3'[t] \to D[\theta_3[t] /. sol, t]} /. t \to tmax][[
    1]] == \lambda * (D[\phi_4, \theta_3[t]] + D[\phi_5, \theta_3[t]]) /. sol /. t \to tmax;
EQ9 = \lambda \neq 0;

NewInitCon = NSolve[{EQ1, EQ2[[1]], EQ3[[1]], EQ4[[1]], EQ5[[1]], EQ6[[1]], EQ7[[1]],
  EQ8[[1]], EQ9}, {x1plus, y1plus, \theta_{1plus}, x2plus, y2plus, \theta_{2plus}, \theta_{3plus}, \lambda}];

PostImpactConditions = {x1[tmax1] == (x1[t] /. sol /. t \to tmax)[[1]],
  y1[tmax1] == (y1[t] /. sol /. t \to tmax)[[1]],

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```

 $\theta_1[t_{\max 1}] = (\theta_1[t] /. \text{sol} /. t \rightarrow t_{\max})[[1]], x_2[t_{\max 1}] =$ 
 $(x_2[t] /. \text{sol} /. t \rightarrow t_{\max})[[1]], y_2[t_{\max 1}] = (y_2[t] /. \text{sol} /. t \rightarrow t_{\max})[[1]],$ 
 $\theta_2[t_{\max 1}] = (\theta_2[t] /. \text{sol} /. t \rightarrow t_{\max})[[1]],$ 
 $\theta_3[t_{\max 1}] = (\theta_3[t] /. \text{sol} /. t \rightarrow t_{\max})[[1]], x_1'[t_{\max 1}] = \text{NewInitCon}[[1, 1, 2]],$ 
 $y_1'[t_{\max 1}] = \text{NewInitCon}[[1, 2, 2]], \theta_1'[t_{\max 1}] = \text{NewInitCon}[[1, 3, 2]],$ 
 $x_2'[t_{\max 1}] = \text{NewInitCon}[[1, 4, 2]], y_2'[t_{\max 1}] = \text{NewInitCon}[[1, 5, 2]],$ 
 $\theta_2'[t_{\max 1}] = \text{NewInitCon}[[1, 6, 2]], \theta_3'[t_{\max 1}] = \text{NewInitCon}[[1, 7, 2]]];$ 
```

(*Release Constraints Post Impact*)

```

(* $\phi_1 = 15 \cos[\theta_3[t]] - ((\text{gWSq1.BR})[[1]]); (*((y_1[t] - 1.5) - (\text{gWS2}[[2,4]]))*)$ ;
 $\phi_2 = 6 - 15 \sin[\theta_3[t]] - ((\text{gWSq1.BR})[[2]]);$ 
 $(*(\text{gWSq2.BL})[[2]] + (1.5 \tan[\theta_3[t]])) - \text{gWS1}[[2,4]]$ ;*)*)
(* $\phi_3 = \theta_2[t] - \theta_3[t]$ ;*)
```

(*SQUARE1*)

```

Eq1 = D[D[L, x1'[t]], t] - D[L, x1[t]] == 0
(* $\lambda_1[t] * D[\phi_1, x_1[t]] + \lambda_2[t] * D[\phi_2, x_1[t]]$ *) (**  $\lambda_3[t] * D[\phi_3, x_1[t]]$ *));
Eq2 = D[D[L, y1'[t]], t] - D[L, y1[t]] == 0
(* $\lambda_1[t] * D[\phi_1, y_1[t]] + \lambda_2[t] * D[\phi_2, y_1[t]]$ *) (**  $\lambda_3[t] * D[\phi_3, y_1[t]]$ *));
Eq3 = D[D[L,  $\theta_1'[t]$ ], t] - D[L,  $\theta_1[t]$ ] == 0
(* $\lambda_1[t] * D[\phi_1, \theta_1[t]] + \lambda_2[t] * D[\phi_2, \theta_1[t]]$ *) (**  $\lambda_3[t] * D[\phi_3, \theta_1[t]]$ *));
```

(*SQUARE2*)

```

Eq4 = D[D[L, x2'[t]], t] - D[L, x2[t]] == 0
(* $\lambda_1[t] * D[\phi_1, x_2[t]] + \lambda_2[t] * D[\phi_2, x_2[t]]$ *) (**  $\lambda_3[t] * D[\phi_3, x_2[t]]$ *));
Eq5 = D[D[L, y2'[t]], t] - D[L, y2[t]] == 0
(* $\lambda_1[t] * D[\phi_1, y_2[t]] + \lambda_2[t] * D[\phi_2, y_2[t]]$ *) (**  $\lambda_3[t] * D[\phi_3, y_2[t]]$ *));
Eq6 = D[D[L,  $\theta_2'[t]$ ], t] - D[L,  $\theta_2[t]$ ] == 0
(* $\lambda_1[t] * D[\phi_1, \theta_2[t]] + \lambda_2[t] * D[\phi_2, \theta_2[t]]$ *) (**  $\lambda_3[t] * D[\phi_3, \theta_2[t]]$ *));
```

(*SEE SAW*)

```

Eq7 = D[D[L,  $\theta_3'[t]$ ], t] - D[L,  $\theta_3[t]$ ] == 0
(* $\lambda_1[t] * D[\phi_1, \theta_3[t]] + \lambda_2[t] * D[\phi_2, \theta_3[t]]$ *) (**  $\lambda_3[t] * D[\phi_3, \theta_3[t]]$ *));
(*Eq8 = D[D[ $\phi_1$ , t], t] == 0;
Eq9 = D[D[ $\phi_2$ , t], t] == 0;*)
(*Eq10 = D[D[ $\phi_3$ , t], t] == 0;*)
```

ELtemp = Solve[Eq1 && Eq2 && Eq3 && Eq4 && Eq5 && Eq6 && Eq7
(*&&Eq8&&Eq9&&Eq10*), {x1''[t], y1''[t], $\theta_1''[t]$, x2''[t],
y2''[t], $\theta_2''[t]$, $\theta_3''[t]$ (*, $\lambda_1[t]$, $\lambda_2[t]$, $\lambda_3[t]$ *)}];

```

EL = {x1''[t] == ELtemp[[1, 1, 2]], y1''[t] == ELtemp[[1, 2, 2]],
 $\theta_1''[t] == ELtemp[[1, 3, 2]], x_2''[t] == ELtemp[[1, 4, 2]], y_2''[t] ==$ 
 $ELtemp[[1, 5, 2]], \theta_2''[t] == ELtemp[[1, 6, 2]], \theta_3''[t] == ELtemp[[1, 7, 2]]};$ 
(*Solve the equations of motion after impact*)
sol = NDSolve[Join[EL, PostImpactConditions],
```

```
{x1[t], y1[t],  $\theta$ 1[t], x2[t], y2[t],  $\theta$ 2[t],  $\theta$ 3[t]}, {t, 0, 10},
Method  $\rightarrow$  {"EventLocator", "Event"  $\rightarrow$  ((y1[t] - 1.5) - (gWS1[[2, 4]])),
"EventAction"  $\Rightarrow$  Throw[tmax = t, "StopIntegration"]];
```

```
(*Updating variables after impact*)
```

```
x1n = Piecewise[{{x1n, 0  $\leq$  t  $\leq$  tmax1}, {sol[[1, 1, 2]], t > tmax1 && t < 10}}];
y1n = Piecewise[{{y1n, 0  $\leq$  t  $\leq$  tmax1}, {sol[[1, 2, 2]], t > tmax1 && t < 10}}];
 $\theta$ 1n = Piecewise[{{ $\theta$ 1n, 0  $\leq$  t  $\leq$  tmax1}, {sol[[1, 3, 2]], t > tmax1 && t < 10}}];
x2n = Piecewise[{{x2n, 0  $\leq$  t  $\leq$  tmax1}, {sol[[1, 4, 2]], t > tmax1 && t < 10}}];
y2n = Piecewise[{{y2n, 0  $\leq$  t  $\leq$  tmax1}, {sol[[1, 5, 2]], t > tmax1 && t < 10}}];
 $\theta$ 2n = Piecewise[{{ $\theta$ 2n, 0  $\leq$  t  $\leq$  tmax1}, {sol[[1, 6, 2]], t > tmax1 && t < 10}}];
 $\theta$ 3n = Piecewise[{{ $\theta$ 3n, 0  $\leq$  t  $\leq$  tmax1}, {sol[[1, 7, 2]], t > tmax1 && t < 10}}];
```

```
(*Getting all Config Variables*)
```

```
fullx1 = PiecewiseExpand[x1n];
fully1 = PiecewiseExpand[y1n];
full $\theta$ 1 = PiecewiseExpand[ $\theta$ 1n];
fullx2 = PiecewiseExpand[x2n];
fully2 = PiecewiseExpand[y2n];
full $\theta$ 2 = PiecewiseExpand[ $\theta$ 2n];
full $\theta$ 3 = PiecewiseExpand[ $\theta$ 3n];
```

In[]:= **(*ANIMATE*)**

```
(*SQUARE1*)
```

```
TL1t[T_] :=
  (((gWSq1.TL) /. x1[t]  $\rightarrow$  fullx1 /. y1[t]  $\rightarrow$  fully1 /.  $\theta$ 1[t]  $\rightarrow$  full $\theta$ 1) /. t  $\rightarrow$  T)[[
    1 ;; 2]];
BL1t[T_] := (((gWSq1.BL) /. x1[t]  $\rightarrow$  fullx1 /. y1[t]  $\rightarrow$  fully1 /.  $\theta$ 1[t]  $\rightarrow$  full $\theta$ 1) /.
  t  $\rightarrow$  T)[[1 ;; 2]];
BR1t[T_] := (((gWSq1.BR) /. x1[t]  $\rightarrow$  fullx1 /. y1[t]  $\rightarrow$  fully1 /.  $\theta$ 1[t]  $\rightarrow$  full $\theta$ 1) /.
  t  $\rightarrow$  T)[[1 ;; 2]];
TR1t[T_] := (((gWSq1.TR) /. x1[t]  $\rightarrow$  fullx1 /. y1[t]  $\rightarrow$  fully1 /.  $\theta$ 1[t]  $\rightarrow$  full $\theta$ 1) /.
  t  $\rightarrow$  T)[[1 ;; 2]];

```

```
(*SQUARE2*)
```

```
TL2t[T_] :=
  (((gWSq2.TL) /. x2[t]  $\rightarrow$  fullx2 /. y2[t]  $\rightarrow$  fully2 /.  $\theta$ 2[t]  $\rightarrow$  full $\theta$ 2) /. t  $\rightarrow$  T)[[
    1 ;; 2]];
BL2t[T_] := (((gWSq2.BL) /. x2[t]  $\rightarrow$  fullx2 /. y2[t]  $\rightarrow$  fully2 /.  $\theta$ 2[t]  $\rightarrow$  full $\theta$ 2) /.
  t  $\rightarrow$  T)[[1 ;; 2]];
BR2t[T_] := (((gWSq2.BR) /. x2[t]  $\rightarrow$  fullx2 /. y2[t]  $\rightarrow$  fully2 /.  $\theta$ 2[t]  $\rightarrow$  full $\theta$ 2) /.
  t  $\rightarrow$  T)[[1 ;; 2]];

```



```
TR2t[T_] := (((gWSq2.TR) /. x2[t] → fullx2 /. y2[t] → fully2 /.  $\theta$ 2[t] → full $\theta$ 2) /.
  t → T) [[1 ;; 2]];
```

```
(*SEE SAW*)
```

```
SS1t[T_] := (((gWMid.{-30/2, -1/2, 0, 1}) /.  $\theta$ 3[t] → full $\theta$ 3) /. t → T) [[1 ;; 2]];
```

```
SS2t[T_] := (((gWMid.{30/2, -1/2, 0, 1}) /.  $\theta$ 3[t] → full $\theta$ 3) /. t → T) [[1 ;; 2]];
```

```
SS3t[T_] := (((gWMid.{30/2, 1/2, 0, 1}) /.  $\theta$ 3[t] → full $\theta$ 3) /. t → T) [[1 ;; 2]];
```

```
SS4t[T_] := (((gWMid.{-30/2, 1/2, 0, 1}) /.  $\theta$ 3[t] → full $\theta$ 3) /. t → T) [[1 ;; 2]];
```

```
(*Spring1*)
```

```
Spring2D[start_, end_, loops_, radius_] :=
```

```
Module[{detail = 40, steps}, steps = detail (loops + .5);
```

```
Translate[Rotate[Line@Table[
  {radius + (Norm[end - start] - 2 radius) a / steps + radius Cos[2 Pi a / detail + Pi],
  radius Sin[2 Pi a / detail]}, {a, 0, steps}], {{1, 0}, end - start}], start]]
```

```
Animate[Show[
```

```
Graphics[{{(*Square1*)Green, Thick, Line[{{TL1t[t], TR1t[t]}, {TR1t[t], BR1t[t]}},
```

```
{BR1t[t], BL1t[t]}, {BL1t[t], TL1t[t]}]}, (*Square2*)Red, Thick,
```

```
Line[{{TL2t[t], TR2t[t]}, {TR2t[t], BR2t[t]}, {BR2t[t], BL2t[t]},
```

```
{BL2t[t], TL2t[t]}]}, (*SeeSaw*)Black, Thick,
```

```
Line[{{SS1t[t], SS2t[t]}, {SS2t[t], SS3t[t]}, {SS3t[t], SS4t[t]},
```

```
{SS4t[t], SS1t[t]}]}, (*SeeSaw Pivot*)Black, Thick,
```

```
Line[{{-4, 0}, {0, 6}, {4, 0}, {-4, 0}}], (*Spring1*)Blue, Thick,
```

```
Spring2D[{15, 0}, SS2t[t], 7, 0.2], (*Spring2*)Blue, Thick,
```

```
Spring2D[{-15, 0}, SS1t[t], 7, 0.2], Black, Thick, Line[{{-20, 0}, {20, 0}}]},
```

```
Axes → False, PlotRange → {{-20, 20}, {-5, 60}}]],
```

```
{t, 0, 7}, AnimationRunning → False,
```

```
AnimationRate → 1]
```